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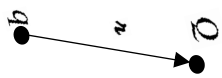
An Institute for IIT-JAM, CUET UG/PG Entrance in Physics & Mathematics

EMT Formula Sheet

ELECTROSTATICS: Study about stationary source charges though the test charge may be moving

Coulomb's law: What is the force on a test charge Q due to source charge q which is at rest a distance r away is

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}\hat{\mathscr{r}}$$



$$\epsilon_0 = 8.85 \times 10^{-12}\frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}.$$

Electric Field: For discrete charge system is

$$\mathbf{E}(\mathbf{r}) \equiv \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i$$

Electric Field: For continuous discrete charge system is

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,dq$$

As $dq \to \lambda\, dl' \sim \sigma\, da' \sim \rho\, d\tau'$. for line surface and volume charge density

For Possible electrostatics Field: $\nabla \times \mathbf{E} = 0$

Electric Flux: Flux through any surface is the number of field lines crossing that surface

$$\Phi_E \equiv \int_{\mathcal{S}} \mathbf{E}\cdot d\mathbf{a}.$$

Gauss Law: Electric flux through any closed surface is equal to the charge enclosed in that surface divided by free space permittivity. For a single point charge q

$$\oint \mathbf{E}\cdot d\mathbf{a} = \frac{1}{\epsilon_0}q.$$

Integral Form of Gauss law

$$\nabla\cdot\mathbf{E} = \frac{1}{\epsilon_0}\rho$$

Differential Forms of Gauss law

Electric field profile for different cases.

(1) E Field due to uniformaly charged solid sphere of radius R
(2) E Field due toinfinitely extended line wire having line charge density λ
(3) Concentric sphere

Electric potential: : Reduce a vector problem to much simpler scalar problem

The line integral of E from point *reference point o* to any point r is

$$V(\mathbf{r}) \equiv -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}.$$

Relation between field and potential

$$\mathbf{E} = -\nabla V.$$

As value of **Del** operator is different in different coordinate systems

Poisson's Equation: Relation between potential and charge density

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

Laplace's equation: For potential V

$$\nabla^2 V = 0.$$

Electrostatic Boundary condition

Perpendicular component of E filed: is discontinuous

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\epsilon_0}\sigma$$

Tangential Components: continuous

$$\mathbf{E}^{\parallel}_{\text{above}} = \mathbf{E}^{\parallel}_{\text{below}}$$

The combined BC formula for E is

$$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$$

Boundary condition in Potential form

The gradient of V inherits the discontinuity in E,

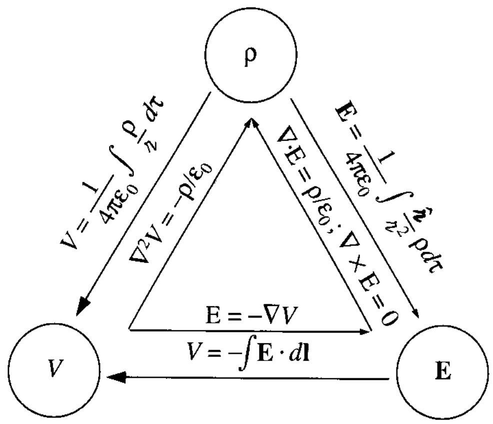
$$\nabla V_{\text{above}} - \nabla V_{\text{below}} = -\frac{1}{\epsilon_0}\sigma\hat{\mathbf{n}},$$

or

$$\frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = -\frac{1}{\epsilon_0}\sigma$$

$\frac{\partial V}{\partial n} = \nabla V \cdot \hat{\mathbf{n}}$ Denotes the normal derivative of V (that is rate of change in the direction perpendicular to the surface)

Three fundamental quantities in electrostatics (E, V, ρ) and relation between them: Triangular diagram



Electrostatics energy density:

$$W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 \, d\tau$$

where, $\frac{\epsilon_0}{2}E^2 =$ energy per unit volume.

Conductor:

(a) Unlimited supply of charges. E=0 inside the conductor (?).
(b) ρ =0 inside conductor. Net charge resides on the surface of conductor. A conductor is an equipotential
(c) Concept of Induced charge

Dielectric: concept of bound charge

- When a piece of dielectric material placed in uniform electric field, it get polarized: separation of charges and creation of dipole
- **Dipole moment**: it is defined as $p = \alpha E$ where α is the polarizability
- **Polarization**: dipole moment per unit volume
- **Volume and Surface bound charge and their relation to polarization vector P**

$$\rho_b = -\nabla \cdot \mathbf{P}, \quad \sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$$

Where ρ_b, σ_b, P volume bound, surface bound charge density and polarization vector respectively

MAGNETOSTAICS: Study about steady Current

Lorentz Force: Sum of electric and magnetic force

$$\mathbf{F} = Q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})].$$

The magnetic force of charge Q moving with velocity v enters in magnetic field B is

$$\boxed{\mathbf{F}_{\text{mag}} = Q(\mathbf{v} \times \mathbf{B}).}$$

Magnetic force for continuous system: Line, Surface and Volume current density:

Magnetic force on a segment of current carrying wire for

(a) Line current $I = \lambda v$ flowing along wire

$$\mathbf{F}_{\text{mag}} = \int (\mathbf{v} \times \mathbf{B})\, dq = \int (\mathbf{v} \times \mathbf{B})\lambda\, dl = \int (\mathbf{I} \times \mathbf{B})\, dl$$

Here $dq \sim \lambda dl \sim \sigma da \sim \rho d\tau$

(b) Surface current $K = \sigma v$ flowing along surface

$$\mathbf{F}_{\text{mag}} = \int (\mathbf{v} \times \mathbf{B})\sigma\, da = \int (\mathbf{K} \times \mathbf{B})\, da$$

(c) Volume current $J = \rho v$ flowing along volume

$$\mathbf{F}_{\text{mag}} = \int (\mathbf{v} \times \mathbf{B})\rho\, d\tau = \int (\mathbf{J} \times \mathbf{B})\, d\tau$$

Relation b/w discrete and continuous system

$$\sum_{i=1}^{n} (\quad) q_i \mathbf{v}_i \sim \int_{\text{line}} (\quad) \mathbf{I}\, dl \sim \int_{\text{surface}} (\quad) \mathbf{K}\, da \sim \int_{\text{volume}} (\quad) \mathbf{J}\, d\tau$$

Note: In Magnetostatics charge density is static

$$\nabla \cdot \mathbf{J} = 0.$$

Biot- Savart law: Magnetic field of steady line current

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\, dl' = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}.$$

where, $\mu_0 = 4\pi \times 10^{-7}\ \text{N/A}^2.$

(a) Magnetic field due to long straight wire carrying steady current I

$$B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1).$$

Case: In case of infinite wire for which $\theta_1 = -\pi/2$ and $\theta_2 = \pi/2$, then

(b) Magnetic field at distance z from the center of circular loop of radius R carrying current I

$$B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{z^2}\right)2\pi R = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$$

Ampere's Law: (a) Integral form

$$\boxed{\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enc}}.}$$

(b) Differential form

$$\boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{J},}$$

Magnetic Vector Potential (A):

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}.}$$

Ampere's law in vector potential form

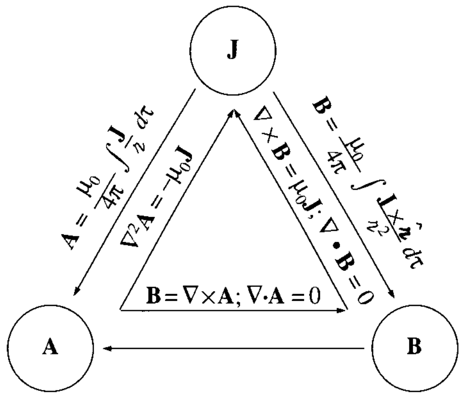
$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

Faraday Law: Integral form

$$\mathcal{E} = \oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

Differential form $$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Triangular diagram: To establishing the relation between fundamental quantities in Magnetostatics magnetic field, current density, mag vector potential (*B*, *J*, *A*)



ELECTRODYNAMICS:

Maxwell's equations

Differential Form — Integral Form

$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ (1) $\quad \oint_s E \cdot dS = \frac{Q_{enc}}{\varepsilon_0}$ (5)

$\nabla \cdot B = 0$ (2) $\quad \oint_s B \cdot dS = 0$ (6)

$\nabla \times E = -\partial B/\partial t$ (3) $\quad \oint_c E \cdot dl = -\frac{\partial \phi_B}{\partial t}$ (7)

$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0\, \partial E/\partial t$ (4) $\quad \oint_c B \cdot dl = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\int_s E \cdot dS$ (8)

Continuity equation

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

Displacement current: time varying electric field produces displacement current

The displacement current in terms of field and potential (as E=V/d)

$$J_d = \frac{\partial(\varepsilon_0 E)}{\partial t} = \frac{\partial D}{\partial t} \quad \text{or} \quad J_d = \varepsilon_0 \frac{\partial E}{\partial t} = \frac{\varepsilon_0}{d}\frac{\partial V}{\partial t}$$

Maxwell Free Space Wave equation in terms of E and B

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \qquad (9)$$

$$\nabla^2 B - \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} = 0 \qquad (10)$$

Note: (1) In free space EM waves travel with the speed of light *c*: *speed of light* $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

(2) Important representation in EMT

$$\nabla -> ik, \nabla^2 -> -k^2, \partial/\partial t -> -i\omega, \frac{\partial^2}{\partial t^2} -> -\omega^2$$

For given profile of one can find the profile (direction) of B as

$$\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega}$$

Poynting's Vector: S is the poynting vector which is the energy per unit time per unit area, transported by the field

$$\mathbf{S} \equiv \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$$

Reflection and Refraction or Transmission of plane wave at Normal incidence: Reflected & transmitted field at the interface of two different media (A) In terms of velocity

$$\tilde{E}_{0_R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) \tilde{E}_{0_I}, \quad \tilde{E}_{0_T} = \left(\frac{2v_2}{v_2 + v_1}\right) \tilde{E}_{0_I}$$

(c) In terms of refractive index as $\frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2}, \; n = \frac{c}{v}$

$$E_{0_R} = \left|\frac{n_1 - n_2}{n_1 + n_2}\right| E_{0_I}, \quad E_{0_T} = \left(\frac{2n_1}{n_1 + n_2}\right) E_{0_I}$$

The reflection & Transmission coefficient of plane wave

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0_R}}{E_{0_I}}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2, \qquad T \equiv \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{E_{0_T}}{E_{0_I}}\right)^2 = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

Where v_1,n_1 and v_2, n_2 are the velocity, refractive index of medium 1 and 2 respectively. I, R and T stands for incident, reflected and transmitted field amplitude or intensity of EM waves.